nth Term Test (also called Divergence Test)

If $\lim_{n\to\infty} a_n$ does not exist or $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=k}^{\infty} a_n$ DIVERGES.
This test can never prove a series converges, the test is inconclusive if $\lim_{n\to\infty} a_n = 0$

Direct Comparison Test (also called Comparison Test)

For the positive series $\sum_{n=k}^{\infty} a_n$ and $\sum_{n=k}^{\infty} b_n$ and arbitrary constant N

If $\sum_{n=k}^{\infty} b_n$ converges and $a_n < b_n$ for all $n > N$ then $\sum_{n=k}^{\infty} a_n$ also CONVERGES.
(If a series is smaller than some finite number it is also some finite number.)

If $\sum_{n=k}^{\infty} b_n$ diverges and $a_n > b_n$ for all $n > N$ then $\sum_{n=k}^{\infty} a_n$ also DIVERGES.
(If a series is bigger than infinity it is also infinity.)

Limit Comparison Test

For the positive series $\sum_{n=k}^{\infty} a_n$ and $\sum_{n=k}^{\infty} b_n$

If $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = c > 0$ then $\sum_{n=k}^{\infty}(a_n)$ and $\sum_{n=k}^{\infty} b_n$ either BOTH CONVERGE or BOTH DIVERGE.
If $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = 0$ and $\sum_{n=k}^{\infty}(b_n)$ converges then $\sum_{n=k}^{\infty} a_n$ also CONVERGES.
If $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \infty$ and $\sum_{n=k}^{\infty}(b_n)$ diverges then $\sum_{n=k}^{\infty} a_n$ also DIVERGES.

Integral Test

For the positive series $\sum_{n=k}^{\infty} a_n$ with $a_n = f(n)$ where $f(n)$ is a continuous, positive, decreasing function of x for all $x > N$. Then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(n)\, dn$ either BOTH CONVERGE or BOTH DIVERGE.

Ratio Test

For the positive series $\sum_{n=k}^{\infty} a_n$

$$\text{Let } \rho = \lim_{n\to\infty} a_{n+1} \div a_n$$

If $\rho > 1$ then the series DIVERGES.
If $\rho < 1$ then the series CONVERGES.
If $\rho < 1$ then the ratio test is inconclusive.

Root Test

For the series $\sum_{n=k}^{\infty} a_n$ where all a_n are positive:

$$\text{Let } \rho = \lim_{n\to\infty} \sqrt[n]{a_n}$$

If $\rho > 1$ then the series DIVERGES.
If $\rho < 1$ then the series CONVERGES.
If $\rho < 1$ then the root test is inconclusive.

Alternating Series Test

The series $\sum_{n=k}^{\infty} a_n$ converges if the following 3 conditions are met:

(1) The terms alternate perfectly (a positive after each negative, a negative after each positive).

(2) $|a_{n+1}| < |a_n|$ for all $n > N$ for some integer N.

(3) $\lim_{n\to\infty} a_n = 0$

Common Series

These are useful for the direct comparison test and limit comparison test.

$\sum_{n=k}^{\infty} \frac{1}{n}$	DIVERGES
$\sum_{n=k}^{\infty} \frac{1}{n^2}$	CONVERGES
$\sum_{n=k}^{\infty} \left(\frac{1}{n}\right)^p$	CONVERGES if $p > 1$ and DIVERGES if $p \leq 1$
$\sum_{n=k}^{\infty} \left(\frac{1}{2}\right)^n$	CONVERGES
$\sum_{n=k}^{\infty} ar^n$	CONVERGES if $\lvert r\rvert < 1$ and DIVERGES if $\lvert r\rvert > 1$